

KHAIRA COLLEGE, KHAIRA, BALASORE

DEPARTMENT OF PHYSICS

QUESTION BANK

UG 4th Sem - CC - VIII

Answer all questions

1- Answer the following :

[1mark]

- a) The value of $(i)^{30} = \underline{\hspace{2cm}}$.
- b) The point at which a function is not analytic is called .
- c) Write the complex form of Fourier integral representation.
- d) The Fourier transform of $e^{-x^2/2}$ simply repeats itself. (True/ False)
- e) If $f(k)$ is the Fourier transform of $f(t)$ the Fourier transform of $f(t \pm a)$ is .
- f) Write heat flow equation in ID.
- g) $L\{t^5\} = \underline{\hspace{2cm}}$.
- h) Define Laplace transform.
- i) Write Polar form of Cauchy – Riemann equations.
- j) Find Laurent series of $f(z) = \frac{1}{1-z}$.
- k) Find the residue of $f(z) = z \cos\left(\frac{1}{z}\right)$ at singular points.
- l) Find Fourier sine Transform of e^{-at} .
- m) What is the value of $A(W)$ if $f(x) = e^{-kx}$?
- n) One dimension Heat conduction is given by $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. (True/ false)
- o) What is the value of $L\{\cos at\}$?
- p) The value of $L^{-1}\left\{\frac{3}{s-2}\right\}$ is .

2- Answer the following (Very short type) :-

[1.5 marks]

- a) Prove that $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$.
- b) Find Polar form of $z = \sqrt{2} - i$.
- c) Obtain Taylor series of $\frac{1}{1+z}$ about $z = 0$.
- d) State convolution theorems of Laplace transform.
- e) Find cube roots of $-11 - 2i$.
- f) Define Laplace transform.
- g) Find Laplace transform of e^{3t} .
- h) Find Fourier sine transform of $\frac{1}{x}$.
- i) Find the complex Fourier transform of $e^{-|x|}$.
- j) Write second shifting theorem of inverse Laplace theorem.
- k) Find the complex conjugate of $\frac{1+2i}{1-i}$.
- l) State Cauchy's Integral theorem.
- m) Find $\oint \frac{e^z}{z^2+9} dz$ inside C if $|z| = 2$ is C.
- n) Find the Taylor's series expansion of $f(z) = \sin z$ about $z = 0$.
- o) Find Fourier sine transform of a^{-at} .
- p) State convolution theorem.
- q) Prove change of scale property of Fourier transform.
- r) Find Laplace's transform of $e^{3t} + e^{-2t}$.
- s) Find the Laplace's transform of $f(t) = t$.
- t) Write Linearity property of Inverse Laplace's transform.

3- Answer the following (Sort type) :-

[2 marks]

- a) Find the location of inverse of $4 - 3i$ in the argand diagram.
- b) State necessary and sufficient condition for a function to be analytic.
- c) Find the analytic function $f(z) = u + iv$, if $v(x,y) = y^2 - x^2$.
- d) Define zeroes and singular point of a complex function.
- e) Find the Fourier cosine integral representation of

$$f(x) = \begin{cases} \sin x, & 0 \leq x \leq \pi \\ 0, & x > \pi \end{cases}$$

- f) Prove shifting property of Fourier transform.
- g) Write down the properties of Dirac delta function.
- h) Find the value of $\int x e^{-3x} \sin x \, dx$.
- i) Find Laplace transform of first derivative of $f(t)$.
- j) Find $f(t)$ whose Laplace transform is $F(s) = \frac{1}{s(s-a)}$.
- k) Find the roots of $z^{1/3}$ if $z = 4 + 3i$.
- l) Write Euler's formula.
- m) Explain the Fourier transform of change of scale property.
- n) Separate $\log e^7$ into real and imaginary part.
- o) Find the residue of $f(z) = \frac{1}{z^2+1}$.
- p) Find the expression of Fourier transform of its derivatives.
- q) Find inverse Laplace transform of $\frac{1}{(s+2)^3}$.
- r) State and prove second shifting property of Laplace transform.
- s) Find Fourier transform of Dirac delta Fourier zero.
- t) State Cauchy's integral formula.

4- Answer the followings (Long type) :-

[6marks]

- a) State and prove Cauchy Riemann condition for analytic function.
- b) State and prove Laurent series expansion.
- c) i) Using Fourier integral. Find the expression for Fourier transform.
ii) Find the Fourier cosine transform of

$$f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

- d) State and prove Fourier integral theorem.
- e) Find the Fourier transform of n th derivative of $f(x)$.
- f) State and prove convolution theorem for Fourier transform.

- g) What is the Periodic function. Find the Laplace's transform of periodic function.
- h) Give a solution of differential equation of an electric circuit 'D' containing R and L in series connected with emf E using Laplace's transform.
- i) Define Cauchy-Reimann conditions in polar form.
- j) State and prove Cauchy's Residue theorem in multiply connected region.
- k) Find the Fourier transform of Gaussian distribution function $f(x) = Ne^{-ax^2}$.
- l) Derive expression for Fourier sine and cosine transform of 1st derivatives.
- m) Find a solution of heat flow equation in ID using Fourier transform.
- n) Find Laplace transform $f(t) = t^n$, $n = 0, 1, 2, \dots$
- o) Using Laplace's transform solve the differential equation $y'' + 2y' + 5y = e^{-x} \sin x$.

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